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Energy-scale dependence of the lepton flavor-mixing matrix

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Abstract. We study an energy-scale dependence of the lepton flavor-mixing matrix in the minimal supersymmetric standard model with the effective dimension-five operators, which give Majorana masses of neutrinos. We analyze the renormalization group equations of the coefficients (κ_{ij}) of these effective operators under an approximation that neglects terms of order κ^2. We find that only $n_{\rm g}-1$ (where $n_{\rm g}$ is the generation number) real independent parameters in the lepton flavor-mixing matrix depend on the energy scale. In particular, all the phases in κ are scale-independent.

1 Introduction

Recent neutrino experiments suggest the existence of flavor mixing in the lepton sector [1]–[5]. Studies of the lepton flavor-mixing matrix, which may be called [6] the Maki–Nakagawa–Sakata (MNS) [7] matrix, have opened up a new era of the flavor physics. When we consider models where the lepton flavor-violating interactions arise from new physics at a high energy scale, it is important to analyze the energy-scale dependence of the MNS matrix to obtain information on the new physics. Many authors analyze their model by using the renormalization group equations (RGE) [8].

In this paper, we study the energy-scale dependence of the MNS matrix in the minimal supersymmetric standard model (MSSM) with effective dimension-five operators, which give Majorana masses of neutrinos. In this model, the superpotential of the lepton–Higgs interactions is

$$\mathcal{W} = {y^{\rm e}}_{ij}(H_{\rm d}L_i)E_j - \frac{1}{2}\kappa_{ij}(H_{\rm u}L_i)(H_{\rm u}L_j)\,. \qquad (1)$$

Here the indices i, j $(= 1 \sim n_{\rm g})$ stand for the generation number. L_i and E_i are chiral superfields of ith generation lepton doublet and right-handed charged lepton, respectively, and $H_{\rm u}$ ($H_{\rm d}$) is that of the Higgs doublet which give Dirac masses to the up- (down-)type fermions. The coefficient κ of the effective dimension-five operator is an $n_{\rm g} \times n_{\rm g}$ complex and symmetric matrix, which gives the neutrino Majorana mass matrix. When we take the diagonal base of the charged-lepton Yukawa coupling $y^{\rm e}$, κ is diagonalized by the MNS matrix. All the elements of κ are naturally small if they are generated effectively by the new physics at a high energy scale M. One of the most attractive scenarios is the seesaw mechanism [9], where the small κ of $O(1/M)$ is generated by the heavy right-handed neutrinos with Majorana masses of $O(M)$.

2 Energy-scale dependence of the MNS matrix

Let us now consider the renormalization of κ. The wave-function renormalization of L_i is given by $L_i^{(0)} = Z_{ij}^{1/2} L_j$ and that of the Higgs doublet is given by $H_{\rm u}^{(0)} = Z_{\rm H}^{1/2} H_{\rm u}$. Then the renormalization of κ_{ij} is written as

$$\kappa_{ij}^{(0)} = \left(Z_{ik}^{-1/2} Z_{jl}^{-1/2} Z_{\rm H}^{-1}\right)\kappa_{kl} \qquad (2)$$

in supersymmetric theories. Here we adopt an approximation to neglect loop corrections of $O(\kappa^2)$, which are sufficiently small because of tiny neutrino masses. If κ is induced by the seesaw mechanism, this approximation is consistent with the neglect of terms of $O(1/M^2)$. Under this approximation, Z_{ik} is diagonal, $Z_{ik} = Z_i\delta_{ik} + O(\kappa^2)$, because there are no lepton flavor-mixing terms except κ in the MSSM Lagrangian. Therefore (2) reduces to

$$\kappa_{ij}^{(0)} = \left(Z_i^{-1/2} Z_j^{-1/2} Z_{\rm H}^{-1}\right)\kappa_{ij}\,. \qquad (3)$$

Equation (3) leads to the RGE

$$\frac{\rm d}{{\rm d}t}\kappa_{ij} = \left(\gamma_i + \gamma_j + 2\gamma_{\rm H}\right)\kappa_{ij}\,, \qquad (4)$$

where t is the scaling parameter, which is related to the renormalization scale μ as $t = \ln\mu$. γ_i and $\gamma_{\rm H}$ are defined as

$$\gamma_i = \frac{1}{2}\frac{\rm d}{{\rm d}t}\ln Z_i\,, \quad \gamma_{\rm H} = \frac{1}{2}\frac{\rm d}{{\rm d}t}\ln Z_{\rm H}\,. \qquad (5)$$

From (4), we obtain the following two consequences:

(1) *None of the phases in κ depend on the energy scale.* Using the notation $\kappa_{ij} \equiv |\kappa_{ij}| \mathrm{e}^{\mathrm{i}\varphi_{ij}}$, we rewrite (4) as

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} \ln \kappa_{ij} &= \frac{\mathrm{d}}{\mathrm{d}t} \ln |\kappa_{ij}| + \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\varphi_{ij} \\ &= \left(\gamma_i + \gamma_j + 2\gamma_{\mathrm{H}}\right) . \end{aligned} \tag{6}$$

Since γ_i, γ_j and γ_{H} are real, (6) implies

$$\frac{\mathrm{d}}{\mathrm{d}t}\varphi_{ij} = 0 . \tag{7}$$

Therefore we can conclude that the arguments of all the elements of κ are not changed by renormalization group (RG) evolutions. We should notice that this result does not necessarily mean that phases of the MNS matrix are independent of the energy scale, as we will see later.

(2) *Only $n_{\mathrm{g}} - 1$ real independent parameters in the MNS matrix depend on the energy scale.* The following combinations of the κ elements,

$$c_{ij}^2 = \frac{\kappa_{ij}^2}{\kappa_{ii}\kappa_{jj}} , \tag{8}$$

are energy scale-independent because

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t} \ln \left(\frac{\kappa_{ij}^2}{\kappa_{ii}\kappa_{jj}} \right) \\ &= 2\frac{\mathrm{d}}{\mathrm{d}t} \ln \kappa_{ij} - \frac{\mathrm{d}}{\mathrm{d}t} \ln \kappa_{ii} - \frac{\mathrm{d}}{\mathrm{d}t} \ln \kappa_{jj} \\ &= 2\left(\gamma_i + \gamma_j + 2\gamma_{\mathrm{H}}\right) - \left(2\gamma_i + 2\gamma_{\mathrm{H}}\right) - \left(2\gamma_j + 2\gamma_{\mathrm{H}}\right) \\ &= 0 . \end{aligned} \tag{9}$$

Since the off-diagonal elements of the κ_{ij} $(i \neq j)$ are given by

$$\kappa_{ij} = c_{ij}\sqrt{\kappa_{ii}\kappa_{jj}} \quad (i \neq j) , \tag{10}$$

their energy-scale dependence can be completely determined by that of the diagonal elements κ_{ii}. One can always take the diagonal elements κ_{ii} to be real by rephasing the lepton fields[1], and once the elements are real, they never become complex, because of the RGE (7). The RGE of κ can hence be governed only by n_{g} equations for the real diagonal elements, κ_{ii}. The diagonal form of y^{e} is held at any energy scale because there is no lepton flavor-violating correction to the RGE of y^{e} up to $O(\kappa)$. Since the overall factor of the matrix κ does not affect the MNS matrix, the energy-scale dependence of the MNS matrix can be determined by $n_{\mathrm{g}} - 1$ real independent parameters. This implies that there are $(n_{\mathrm{g}} - 1)^2 = n_{\mathrm{g}}(n_{\mathrm{g}} - 1) - (n_{\mathrm{g}} - 1)$ scale-independent relations among the MNS matrix elements, because the MNS matrix generally has $n_{\mathrm{g}}(n_{\mathrm{g}} - 1)$ real independent parameters when neutrinos are Majorana fermions.

[1] In the base where the charged-lepton Yukawa matrix is diagonalized, phases of κ_{ii} can be absorbed by the field redefinitions as $L_i \to \mathrm{e}^{-\mathrm{i}\varphi_{ii}/2} L_i$ and $E_i \to \mathrm{e}^{\mathrm{i}\varphi_{ii}/2} E_i$, where $\varphi_{ii} = \arg(\kappa_{ii})$

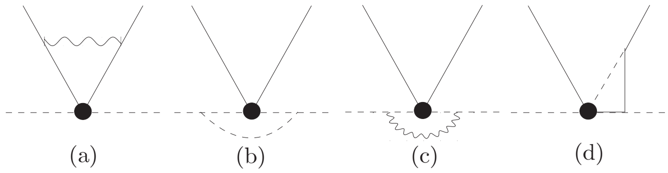

Fig. 1a–d. In the standard model, there are four one-loop diagrams that contribute to the vertex correction of κ_{ij}. Solid lines denote the charged leptons, dotted lines denote the Higgs, and the curved line denotes the Z boson

Before closing, let us comment on the validity of our theorems in the standard model (SM). In non-SUSY models, nonzero vertex corrections contribute to additional terms $(\gamma_{ij}^{\mathrm{v}})$ in the parentheses of the right-hand side of (4), which are generally neither real nor flavor-independent. However, in the SM, one can explicitly show that these corrections are written as [10]

$$\gamma_{ij}^{\mathrm{v}} = \gamma_i^{\mathrm{v}} + \gamma_j^{\mathrm{v}} + \gamma^{\mathrm{v}} \tag{11}$$

at the one-loop level, where all of γ in (11) are real. The diagrams contributing to (11) are shown in Fig. 1. The diagrams (a), (b), and (c) contribute to γ^{v} because these contributions are independent of the flavors. Since the contribution of the diagram (d) is proportional to $(y_i^{\mathrm{e}})^2\kappa_{ij}$ or $(y_j^{\mathrm{e}})^2\kappa_{ij}$, it depends on only one of the flavor indices, i or j. Thus it is clear that our theorem is valid in the SM at the one-loop level when the following substitutions are made in (4): $\gamma_i \to \gamma_i + \gamma_i^{\mathrm{v}}$, $\gamma_j \to \gamma_j + \gamma_j^{\mathrm{v}}$ and $\gamma_{\mathrm{H}} \to \gamma_{\mathrm{H}} + \gamma^{\mathrm{v}}$.

3 An example of the three-generation case

Let us show an example for the case of three generations ($n_{\mathrm{g}} = 3$). The matrix κ can be parameterized as

$$\kappa = \kappa_{33} \begin{pmatrix} r_1 & c_{12}\sqrt{r_1 r_2} & c_{13}\sqrt{r_1} \\ c_{12}\sqrt{r_1 r_2} & r_2 & c_{23}\sqrt{r_2} \\ c_{13}\sqrt{r_1} & c_{23}\sqrt{r_2} & 1 \end{pmatrix} , \tag{12}$$

where

$$r_i \equiv \frac{\kappa_{ii}}{\kappa_{33}} , \quad (i = 1, 2) . \tag{13}$$

The complex parameters c_{ij} are energy scale-independent. There are nine degrees of freedom in the complex 3×3 matrix κ: three complex constants c_{ij} $(i \neq j)$ and three energy scale-dependent real parameters r_1, r_2, and κ_{33}. Since we take a base where y^{e} has a diagonal form at any energy scale, and since κ_{33} does not affect the MNS matrix, only two parameters, r_1 and r_2, determine the energy-scale dependence of the MNS matrix. The MNS matrix has $3(3 - 1) = 6$ real independent parameters, three mixing angles and three phases when neutrinos are Majorana fermions. On the other hand, the energy dependence of the κ is governed by only two equations, which are the RGE of r_1 and r_2. It then follows that there are

$(3-1)^2 = 4$ scale-independent relations among the MNS matrix elements.

Here we roughly estimate the energy-scale dependence of r_1 and r_2 in (13) by using the one-loop RGEs in the MSSM [10,11]. We can easily show that the RGE of r_i is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\ln r_i = \frac{\mathrm{d}}{\mathrm{d}t}\ln\frac{\kappa_{ii}}{\kappa_{33}} = -\frac{1}{8\pi^2}\left(y_\tau^2 - y_i^2\right), \quad (i=1,2), \tag{14}$$

where y_τ and y_i are the Yukawa couplings of τ and an ith ($i=1,2$) generation charged lepton, respectively, in our base where the charged-lepton Yukawa matrix y^{e} is diagonal. Neglecting the small energy-scale dependence of y_i in the MSSM, we see the magnitude of the right-hand side in (14) is roughly given by $y_\tau^2(m_Z)/8\pi^2 = O(10^{-6})/\cos^2\beta$, where the Z-boson mass m_Z represents the weak scale, and $\tan\beta = \langle H_{\mathrm{u}}\rangle/\langle H_{\mathrm{d}}\rangle$ is the ratio of the two vacuum expectation values. This means that the two elements, r_1 and r_2, change only a little between the weak scale and the scale where the effective operator κ appears.

We should stress here that this fact does not necessarily result in the tiny energy dependence of the MNS matrix. We can explicitly see the significant RGE corrections of the MNS matrix in some situations [10,11]. In [11], drastic change of the MNS matrix by the RGE was obtained when neutrinos of the second and the third generations had masses of O(eV) with $\delta m^2_{23} \simeq 3\times10^{-3}$ (eV^2) [3]. This situation corresponds to the case of $r_1 \sim |c_{12}| \sim |c_{13}| \sim 0$, $r_2 \sim 1$, and $|c_{23}| \ll 1$ in (11), where the slight change of r_2 near unity can induce the maximal mixing of the second and the third generations in the MNS matrix.

4 Summary and discussion

In this paper, we studied the energy-scale dependence of the MNS matrix in the MSSM with the effective dimension-five operator that gives rise to the neutrino Majorana masses. The coefficient of the dimension-five operator, κ, has only small enough components that one can neglect corrections of $O(\kappa^2)$ in the RGEs. Under this approximation, we found that none of the phases in κ depend on the energy scale, and that only $n_{\mathrm{g}}-1$ real independent parameters in the $n_{\mathrm{g}}\times n_{\mathrm{g}}$ MNS matrix depend on the energy scale. This implies that there are $(n_{\mathrm{g}}-1)^2 = n_{\mathrm{g}}(n_{\mathrm{g}}-1)-(n_{\mathrm{g}}-1)$ scale-independent relations among the MNS matrix elements, because the MNS matrix generally has $n_{\mathrm{g}}(n_{\mathrm{g}}-1)$ real independent parameters when neutrinos are Majorana fermions. These results may be helpful for the study of lepton flavor physics and the search for new physics at high energies.

Finally, we discuss the validity of our theorem in other models with the effective dimension-five operators. In addition to the smallness of the coefficient matrix elements κ_{ij}, the following two assumptions are needed to obtain the above results: (i) The supersymmetry (SUSY) is needed for (2) to be obtained,; (ii) for (2) to reduce to (3), the model should not have additional lepton flavor- violating terms. Hence our theorem applies in SUSY models without explicit flavor violating terms in the Lagrangian, e.g., in the next-to-minimal SUSY standard model (NMSSM). On the other hand, we cannot directly apply our analysis to the SM or other non-SUSY models, because nonzero vertex renormalization generates additional terms in the right-hand side of (4) which are generally neither real nor flavor-independent. Nevertheless, one can explicitly show that these terms are real and are written as (11) at the one-loop level in the SM [10]. Therefore our theorem applies in the SM at the one-loop level.

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Note added in proof: After the present paper was completed, we received a paper by J. Ellis and S. Lola [12], who also studied the relation between the renormalization effects and the MNS matrix in the special case.